Conservative Mesh Decimation for Collision Detection and Occlusion Culling

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About This Work

Occlusion Culling

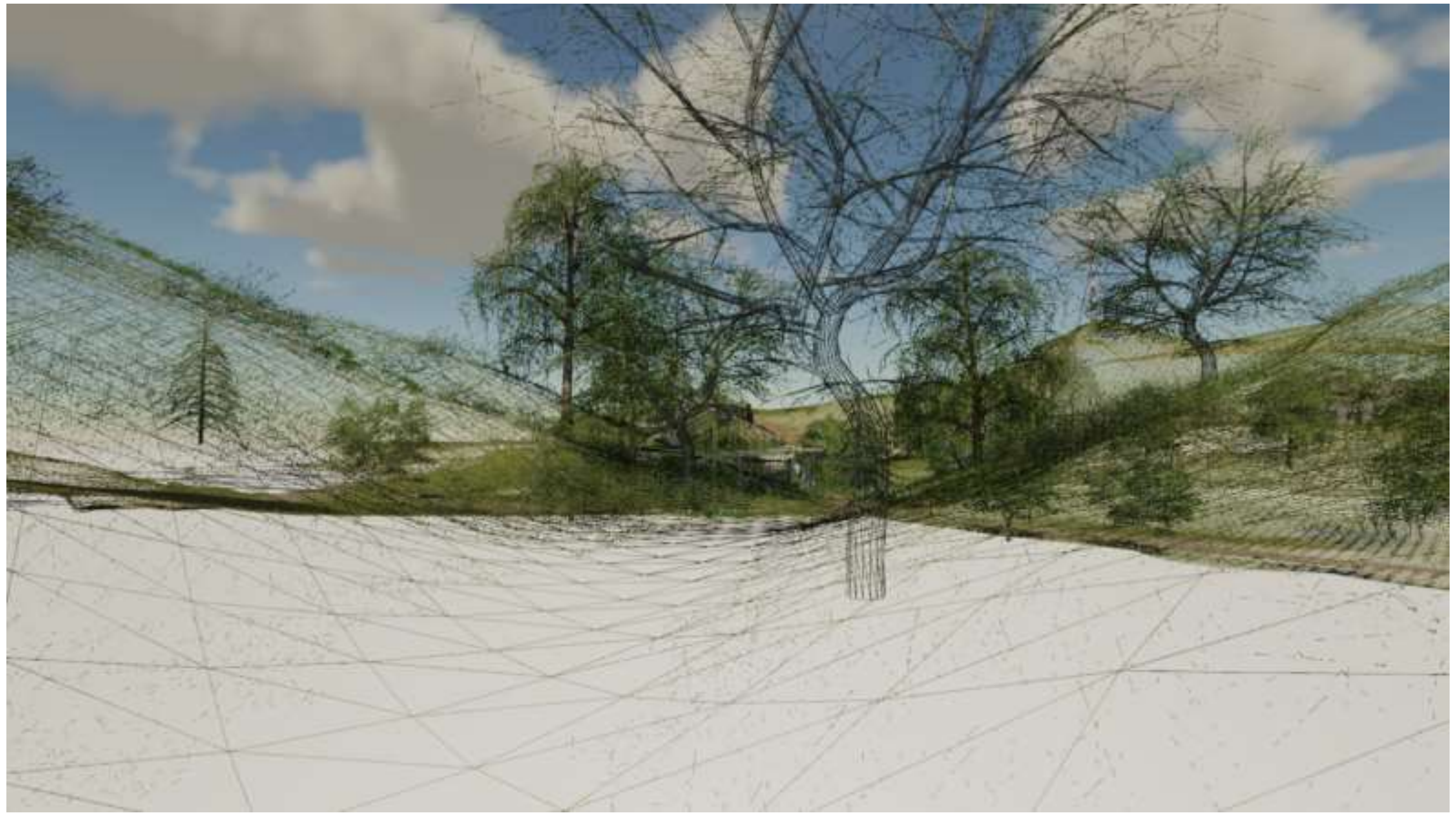
- Farming Simulator uses *depth culling* for accelerated rendering of complex scenes.
- Intel's *MaskedOcclusionCulling* library is used for depth tests on SIMD-capable CPUs.
- Potentially occluding objects are drawn as low-poly meshes into a hierarchical depth-buffer.
- Occluders for terrain patches are generated by conservative mesh decimation.

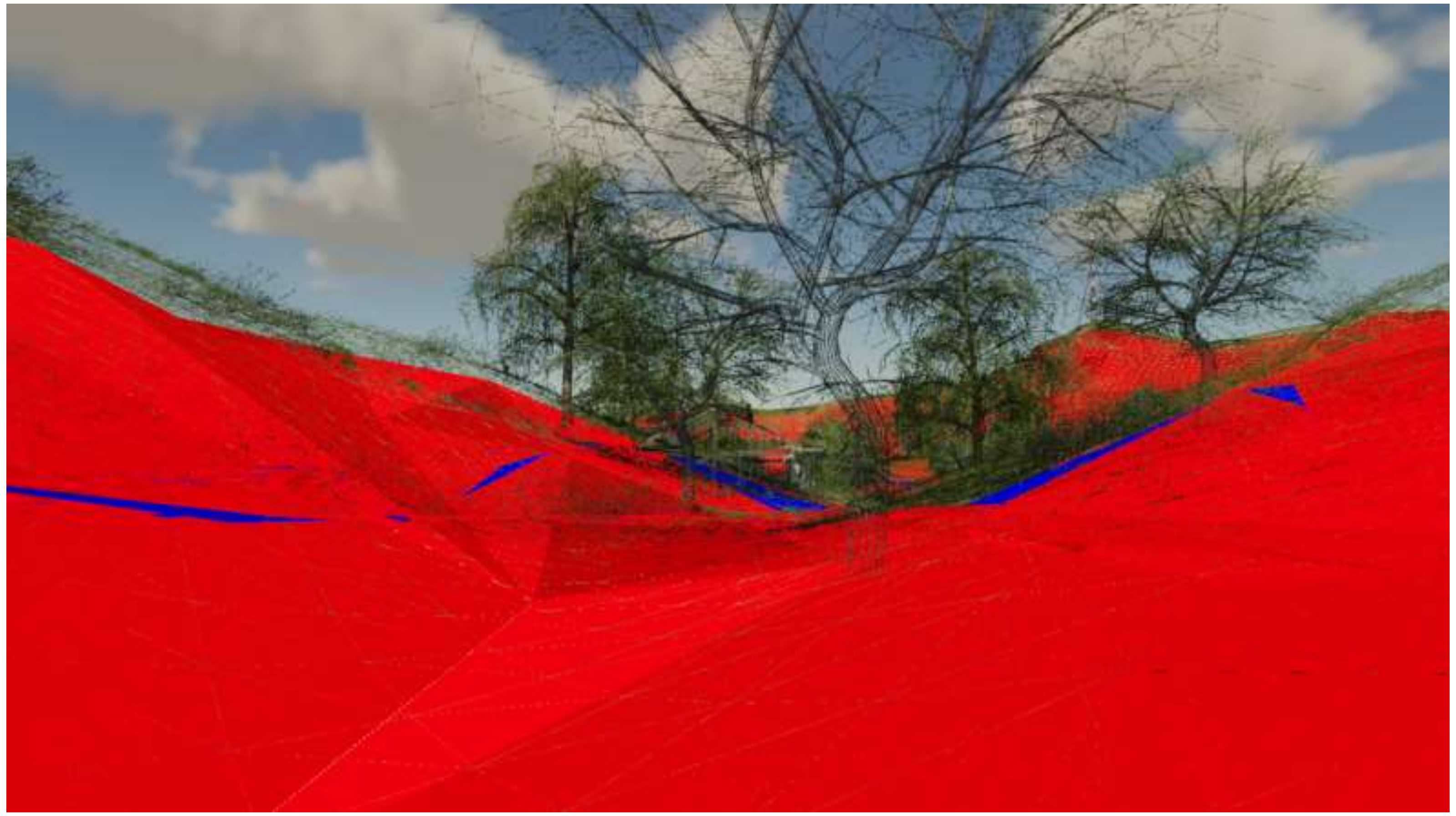
Terrain in Farming Simulator

Terrain in Farming Simulator

- Terrain rendering uses 1025x1025 height maps (2M triangles).
- Height maps are dynamic. Player can modify terrain locally, e.g. dig a ditch.
- Each height map is subdivided into 16x16 patches from which occluders are generated.
- Occluders of modified patches are updated and stitched back to their neighbors.

Terrain Occluder Patches





Edge Contraction

- Edge is contracted to a single vertex.
- Vertex position is chosen such that error is minimized.

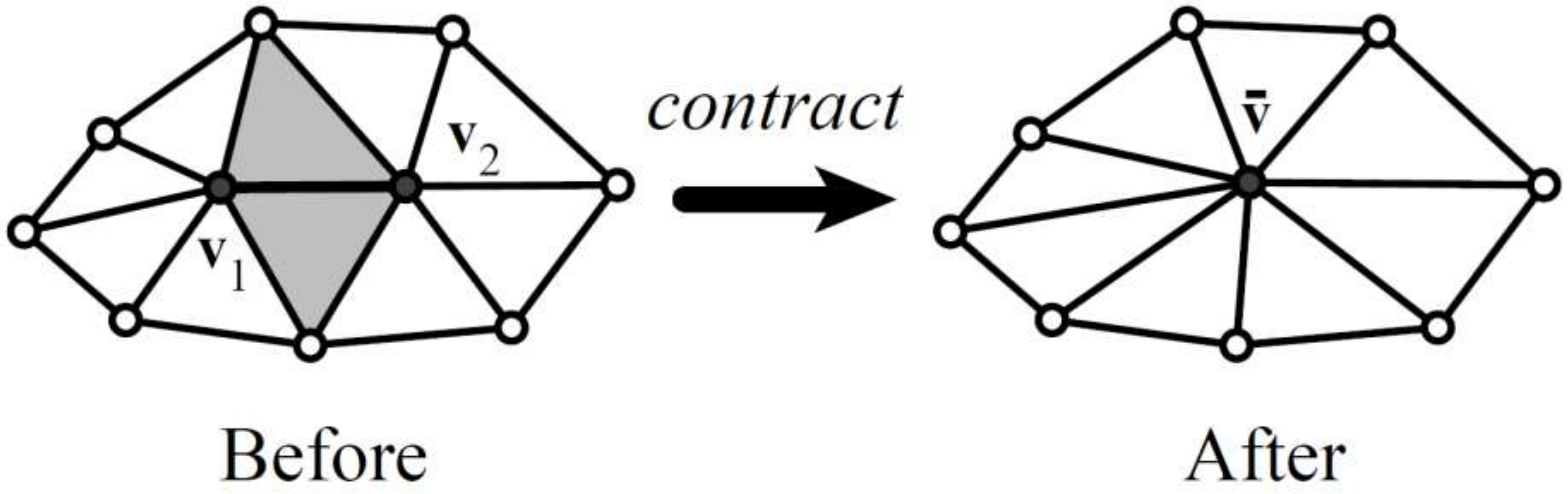


Image: M. Garland and P.S. Heckbert, SIGGRAPH '97

Hausdorff Distance

- The maximum distance from a point of a mesh to the closest point of the other mesh.
- Expresses how well a mesh resembles a target mesh.

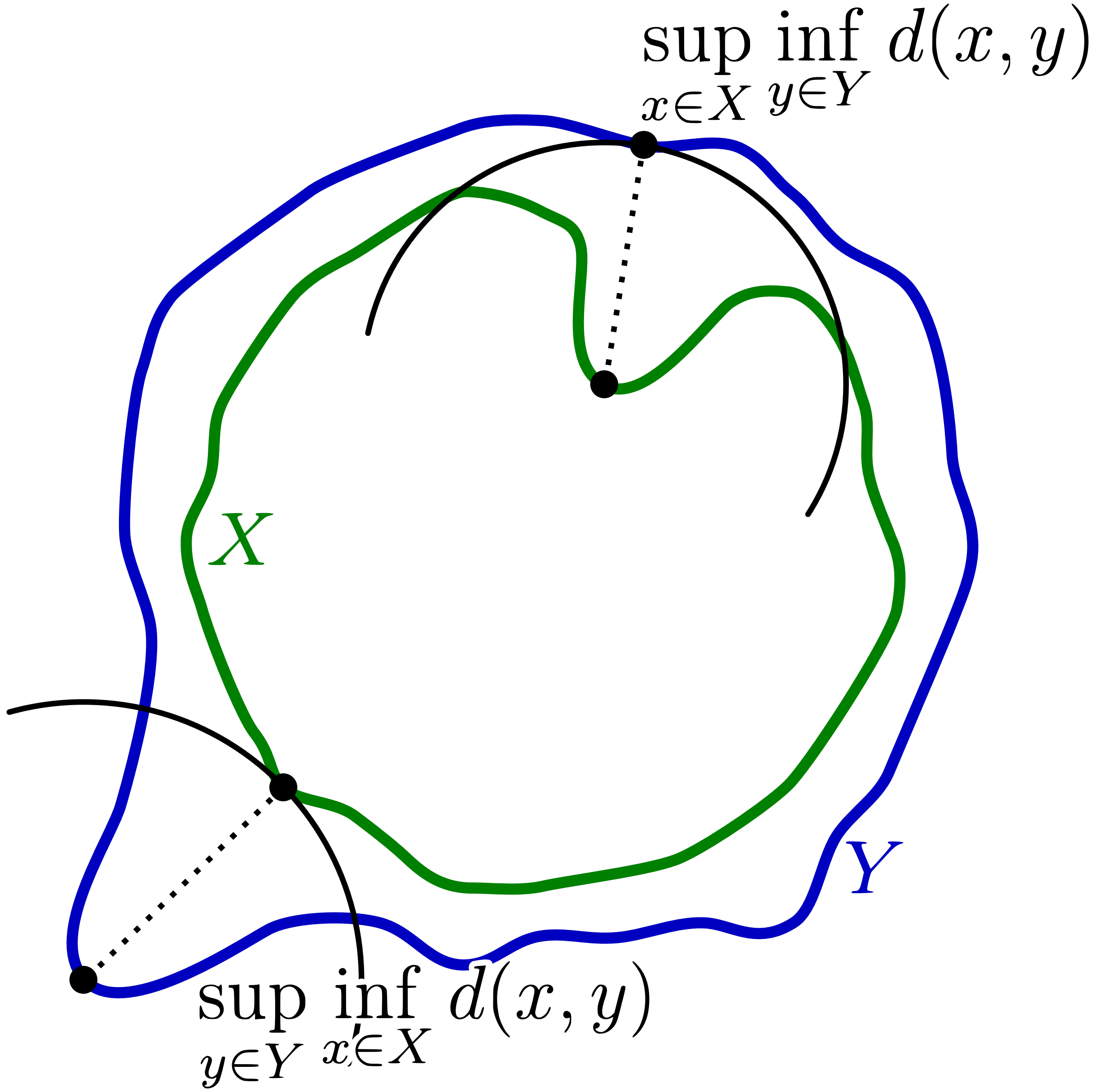


Image: Wikipedia, Creative Commons

Quadric Error Metric

- Computation of Hausdorff distance is expensive.
- Quadric Error Metric (QEM) expresses the distance to the original mesh local to each (new) vertex.
- QEM offers an upper bound for the Hausdorff distance and is cheaper to compute.

Plane Equation

- A plane has equation $ax + by + cx + d = 0$, or rather, $\mathbf{n} \cdot \mathbf{x} + d = 0$, where $\mathbf{n} = (a, b, c)$ normal to the plane, and $\mathbf{x} = (x, y, z)$ a point.

- If $\mathbf{n}$ is normalized ($a^2 + b^2 + c^2 = 1$) then $\mathbf{n} \cdot \mathbf{x} + d$ is the signed distance from $\mathbf{x}$ to the plane.

Homogeneous Coordinates

- In matrix form, the signed distance is expressed as: $[a\ b\ c\ d]\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \mathbf{p}^\top \mathbf{x}$.
- We need the absolute distance as metric.
- Absolute value is awkward so we use square value: $(\mathbf{p}^\top \mathbf{x})^2 = (\mathbf{p}^\top \mathbf{x})^\top \mathbf{p}^\top \mathbf{x} = \mathbf{x}^\top \mathbf{p}\mathbf{p}^\top \mathbf{x}$

Quadratic Form

- Matrix $\mathbf{Q} = \mathbf{p}\mathbf{p}^\top$, a.k.a. the *outer product* of $\mathbf{p}$ with itself, looks like this:

$$\mathbf{Q} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} [a\ b\ c\ d] = \begin{bmatrix} a^2 & ab & ac & ad \\ ba & b^2 & bc & bd \\ ca & cb & c^2 & cd \\ da & db & dc & d^2 \end{bmatrix}$$

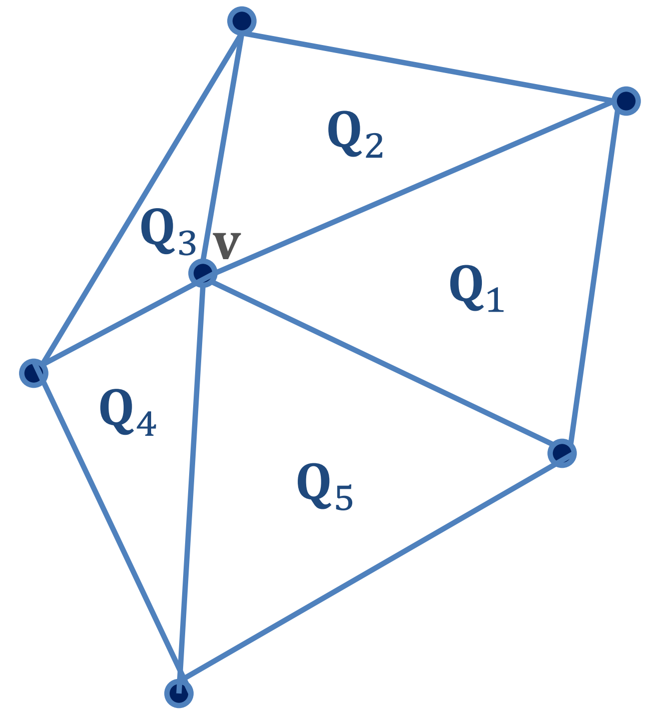
- The squared distance to the plane is $\mathbf{x}^\top\mathbf{Q}\mathbf{x}$.

Positive Semi-definite Matrix

- It follows that $\mathbf{x}^\top \mathbf{Q}\mathbf{x} \geq 0$ for each point $\mathbf{x}$.
- Such matrix is called *positive semi-definite*.
- For $\mathbf{A}$ and $\mathbf{B}$ positive semi-definite matrices, the sum $\mathbf{A} + \mathbf{B}$ is also positive semi-definite.
- Partial ordering: $\mathbf{A} \geq \mathbf{B}$ if $\mathbf{A} - \mathbf{B}$ is positive semi-definite.
- Obviously, $\mathbf{x}^\top \mathbf{A}\mathbf{x} \geq \mathbf{x}^\top \mathbf{B}\mathbf{x}$ only if $\mathbf{A} \geq \mathbf{B}$.

Quadric Error Metric (QEM)

- The sum of matrices $\mathbf{Q}_i$ over all planes i of faces incident to vertex $\mathbf{v}$ bounds the squared Hausdorff distance for points local to $\mathbf{v}$.



$$\mathbf{Q}_\mathbf{v} = \mathbf{Q}_1 + \cdots + \mathbf{Q}_5$$

Quadric Error Metric (Cont'd)

- The set of points $\mathbf{x}$, for which $\mathbf{x}^\top \mathbf{Q} \mathbf{x} = \epsilon^2$, is a *quadric* surface (ellipsoid, elliptical cylinder, or pair of planes).
- Minimum is center (point, line, or plane).

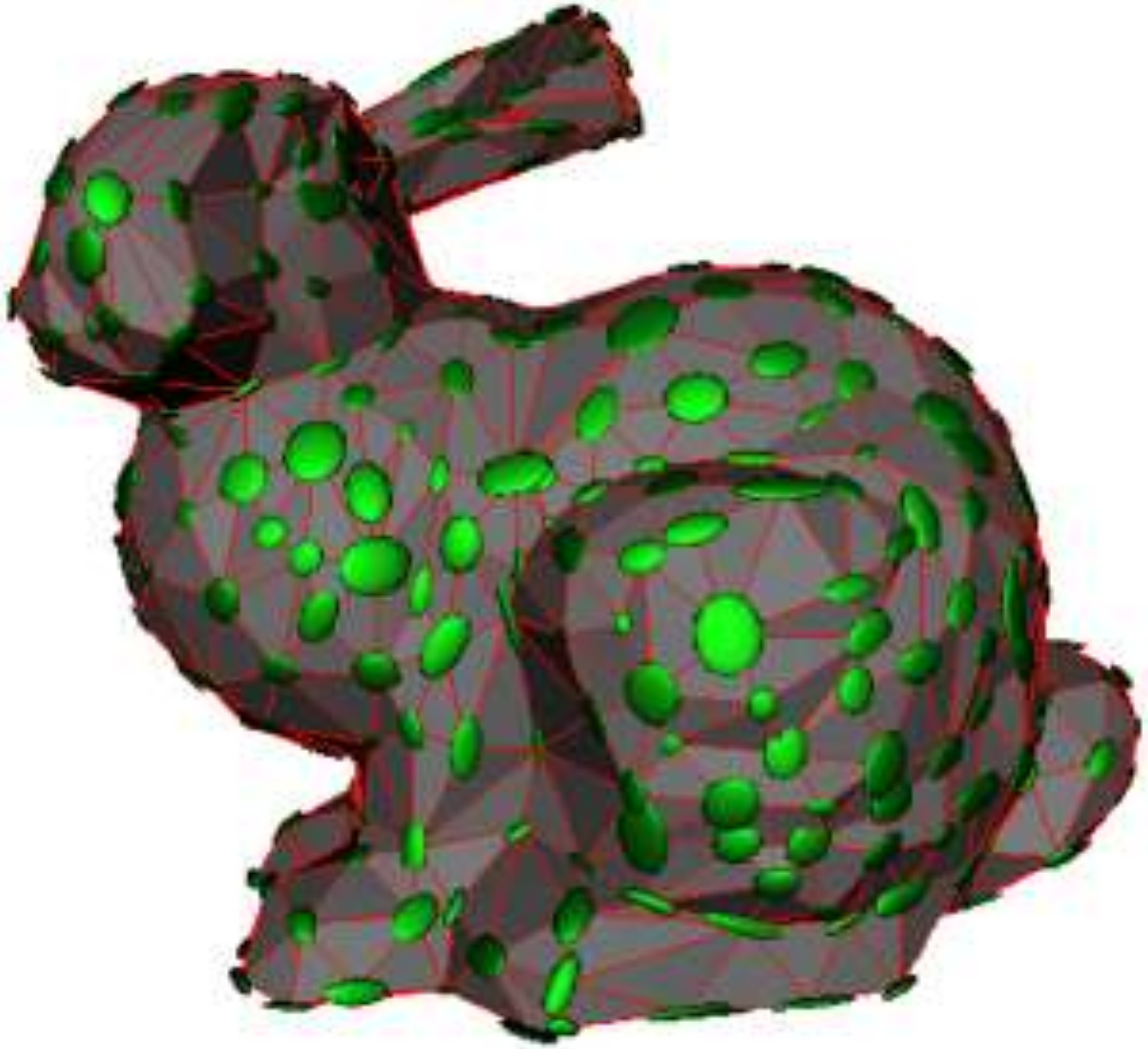
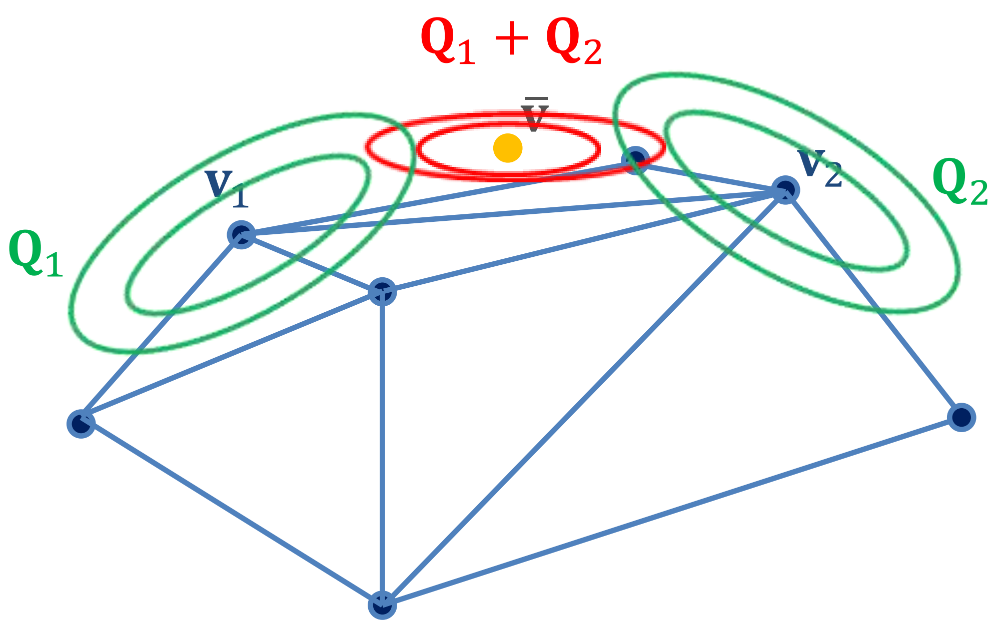


Image: M. Garland and P.S. Heckbert, SIGGRAPH '97

Garland-Heckbert Algorithm

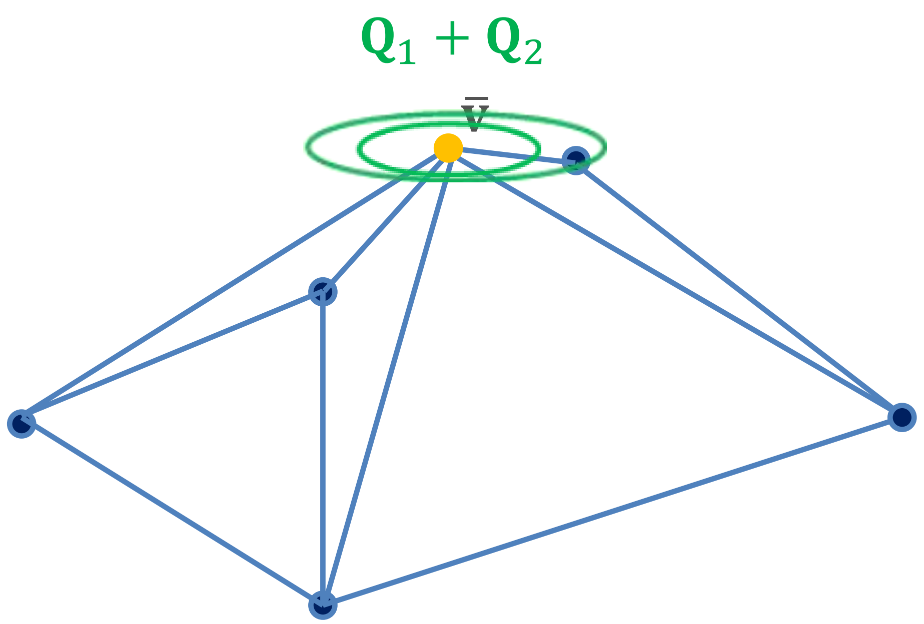
- For each edge $\mathbf{v}_1\mathbf{v}_2$, compute the position $\mathbf{x}$ that minimizes $\mathbf{x}^\top(\mathbf{Q}_1 + \mathbf{Q}_2)\mathbf{x}$.
- This will be the position of the new vertex $\bar{\mathbf{v}}$ after contraction.
- Queue edges prioritizing on the (squared) error of the new vertex position.



Garland-Heckbert Algorithm

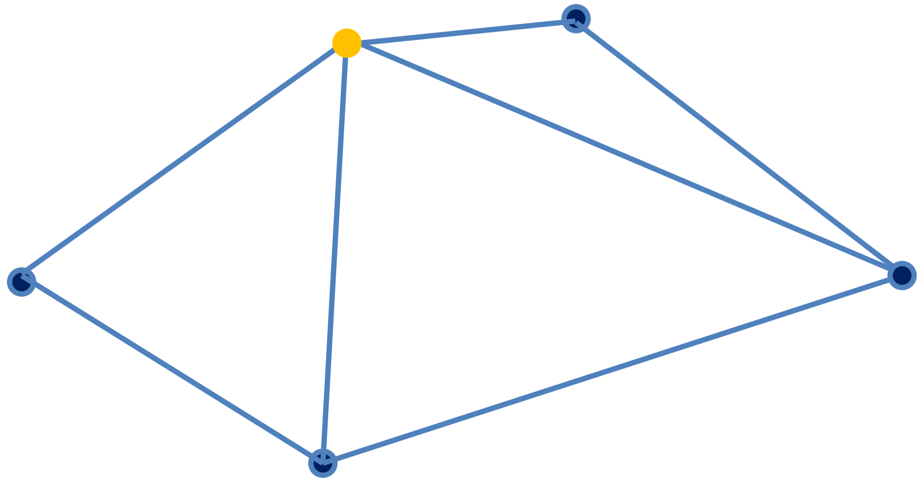
- Contract the least-error edge and set $\mathbf{Q}_1 + \mathbf{Q}_2$ as new QEM of the new vertex .

- Recompute the contraction errors for all edges incident to the new vertex, and update their queue positions.



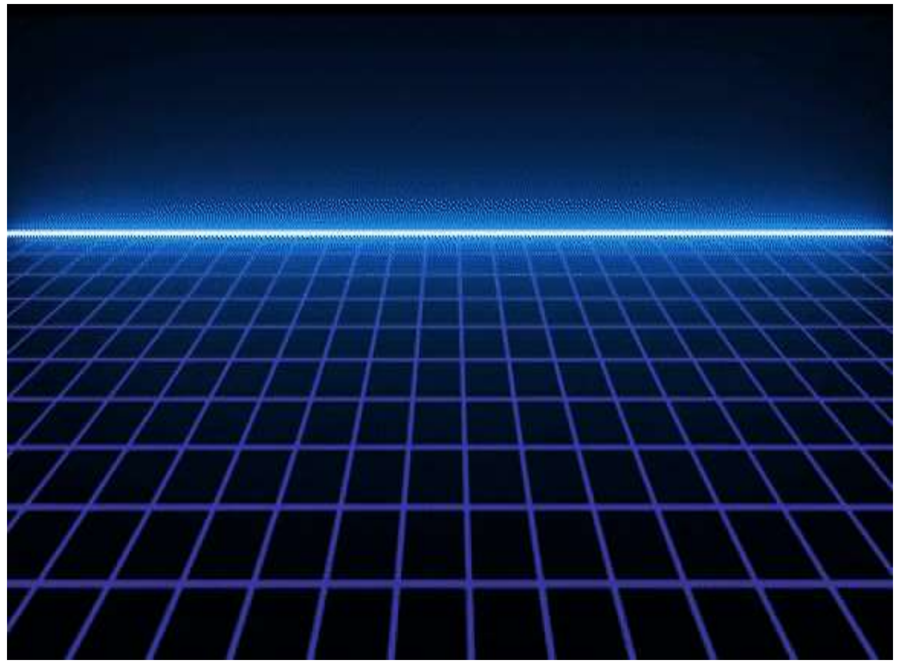
Garland-Heckbert Algorithm

- Continue until the desired error or face count has been reached.
- The final error is an upper bound for the actual error.
- The actual error may be a lot smaller.



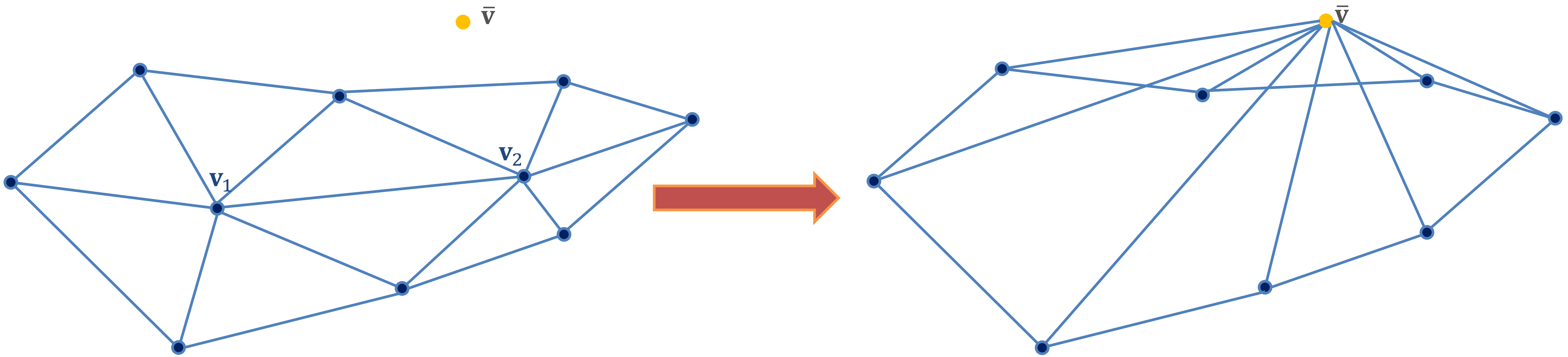
Problem #1: Multiple Solutions

- System has a unique solution for ellipsoidal QEMs only! Solver fails if minimum is a line or a plane.
- Example: straight edge
  flat plane
- Forcing a solution using *pseudo-inverse* is no good. (Prefers solution closest to origin).



Problem #2: Face Flips

- New vertex lies beyond the faces incident to the contracted edge.



Problem #2: Face Flips

- Contracting to a vertex that lies beyond the faces incident to the edge results in flipped faces.
- Detect face flips by testing normals of all new faces against old face normals.
- Reject edge if for any incident face the normals are opposite.

Solution: Rubber Band

- Both problems are mitigated by adding an error component that slightly pulls the new vertex to its original vertices.
- The squared distance to a vertex position $\mathbf{p}$ is expressed as $\mathbf{x}^\top \mathbf{P} \mathbf{x}$, where

$$\mathbf{P} = \begin{bmatrix} I_3 & -\mathbf{p} \\ -\mathbf{p} & \|\mathbf{p}\|^2 \end{bmatrix}$$

  a 4x4 positive semi-definite matrix.

Solution: Rubber Band (cont'd)

- The initial QEM of a vertex is computed as

  $\mathbf{Q}_{\mathbf{v}} = \mathbf{Q}_1 + \cdots + \mathbf{Q}_n + \mathbf{P}\omega$, where $0 < \omega \ll 1$.

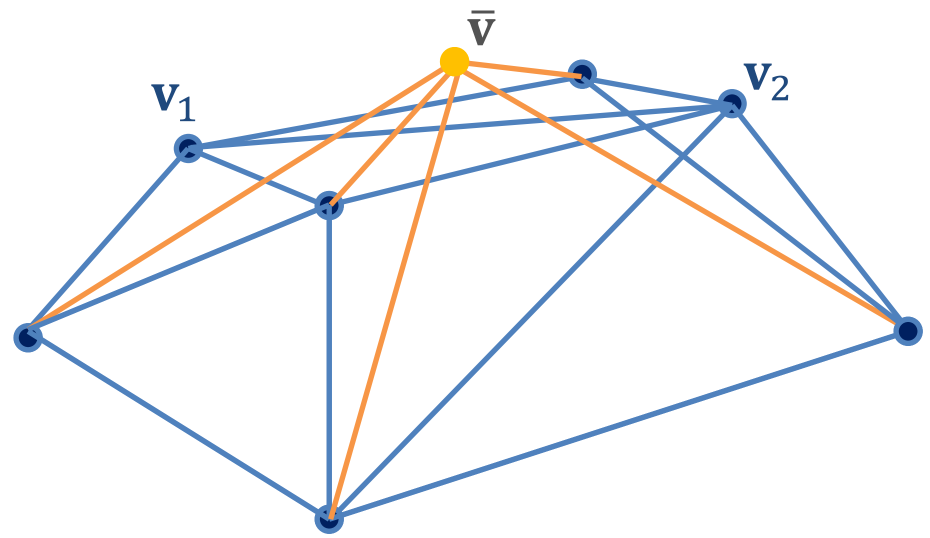
  "The sum of the squared distances to each of its incident faces plus a tiny fraction of the squared distance to the vertex position"

Solution: Rubber Band (cont'd)

- This results in far less singularities in the solver.
- The minimum position is pulled slightly closer to the contracted edge, resulting in fewer edge rejections due to face flips.
- Generated triangles are generally 'fatter', which is helpful in many applications.

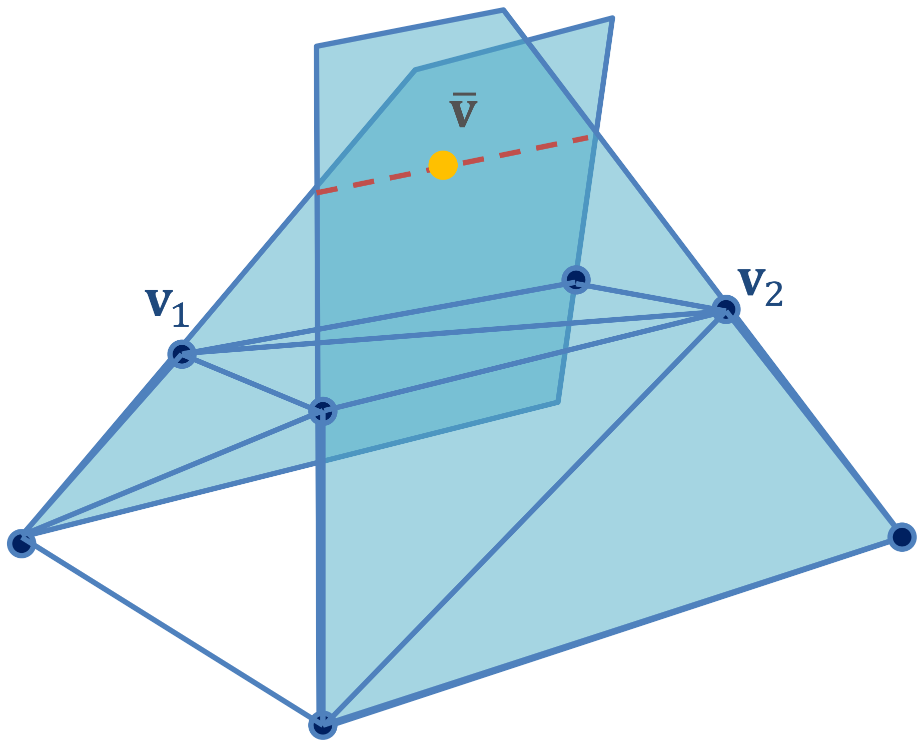
Conservative Mesh Decimation

- Contracting $\mathbf{v}_1\mathbf{v}_2$ to minimal point $\bar{\mathbf{v}}$ creates a mesh that does not bound the original mesh.
- Neither is the new mesh bounded by the original mesh.
- How do we decimate the mesh conservatively?



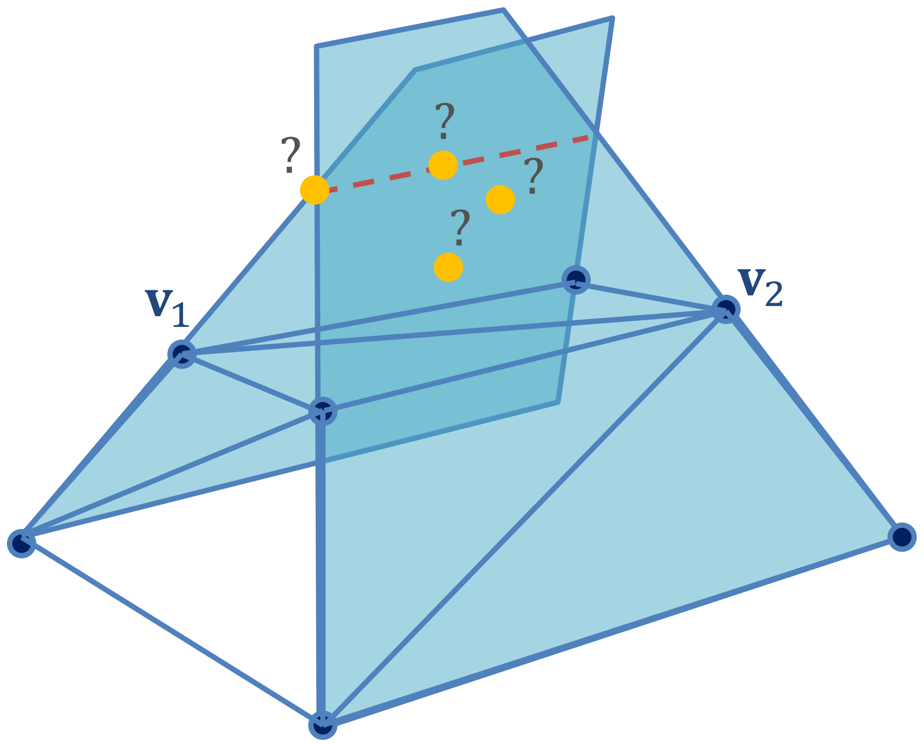
Conservative Mesh Decimation

- For a *bounding mesh*, the new vertex $\bar{\mathbf{v}}$ should not lie *behind* any plane supporting a face incident to the edge.
- For an *occluder*, the new vertex should not lie *in front of* any such plane.
- Such $\bar{\mathbf{v}}$ is called *conservative*.



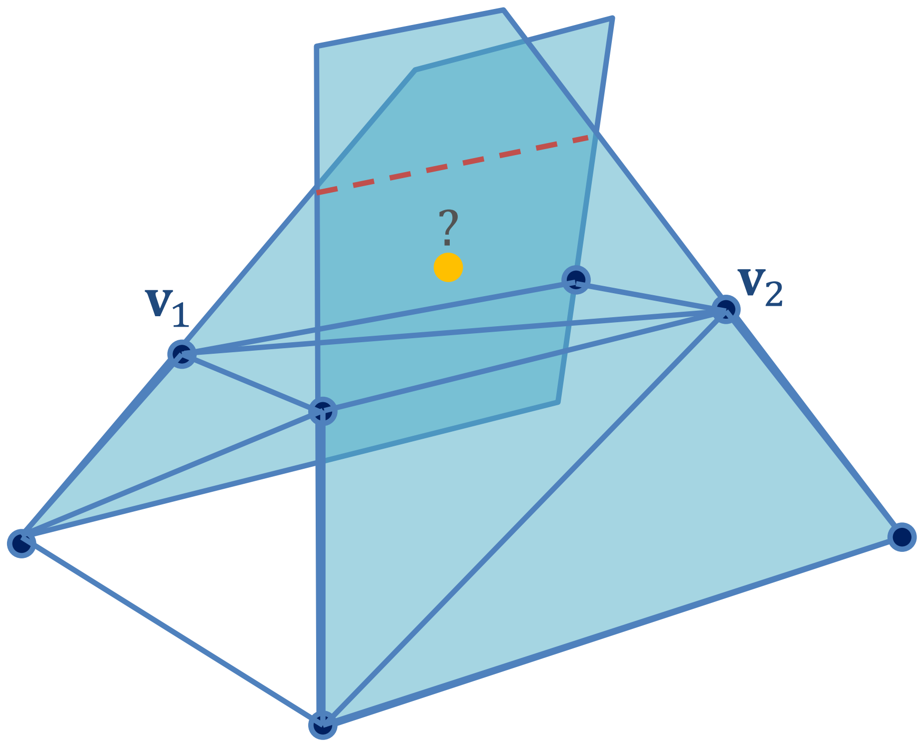
Conservative Mesh Decimation

- The minimal conservative point could lie on zero to three supporting planes.
- Requires solvers for the minimal point in space, on a plane, on a line, and the point of intersection of three planes.



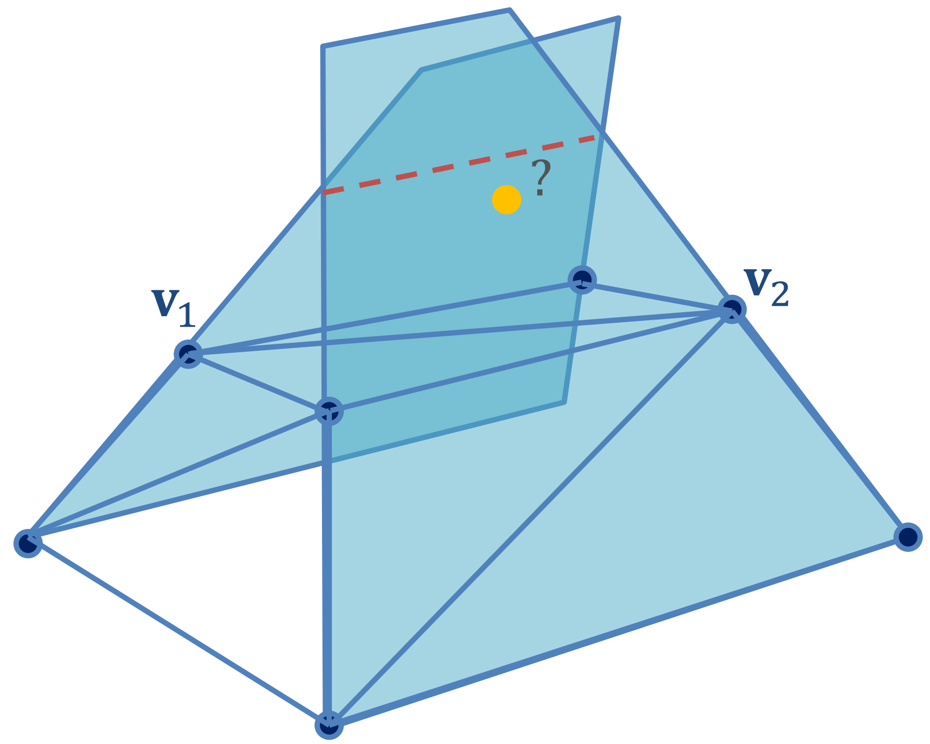
Bounding Mesh Algorithm

- If $\mathbf{Q}_1 + \mathbf{Q}_2$'s minimum point is conservative, it is the new $\bar{\mathbf{v}}$.



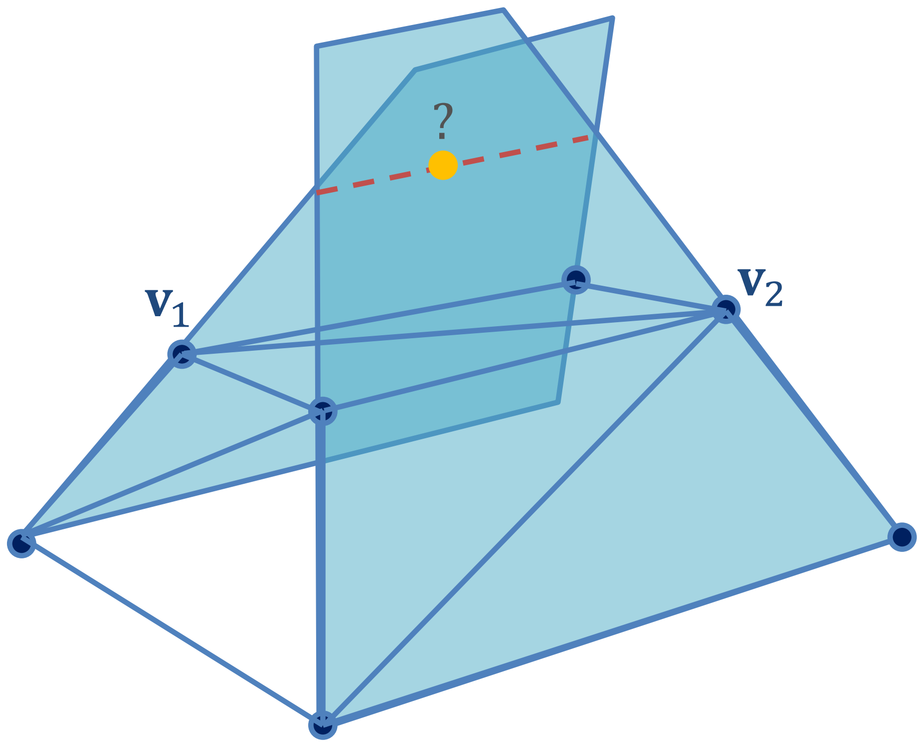
Bounding Mesh Algorithm

- If $\mathbf{Q}_1 + \mathbf{Q}_2$'s minimum point is conservative, it is the new $\bar{\mathbf{v}}$.
- Otherwise, $\bar{\mathbf{v}}$ is the closest conservative minimum point on a plane, or…



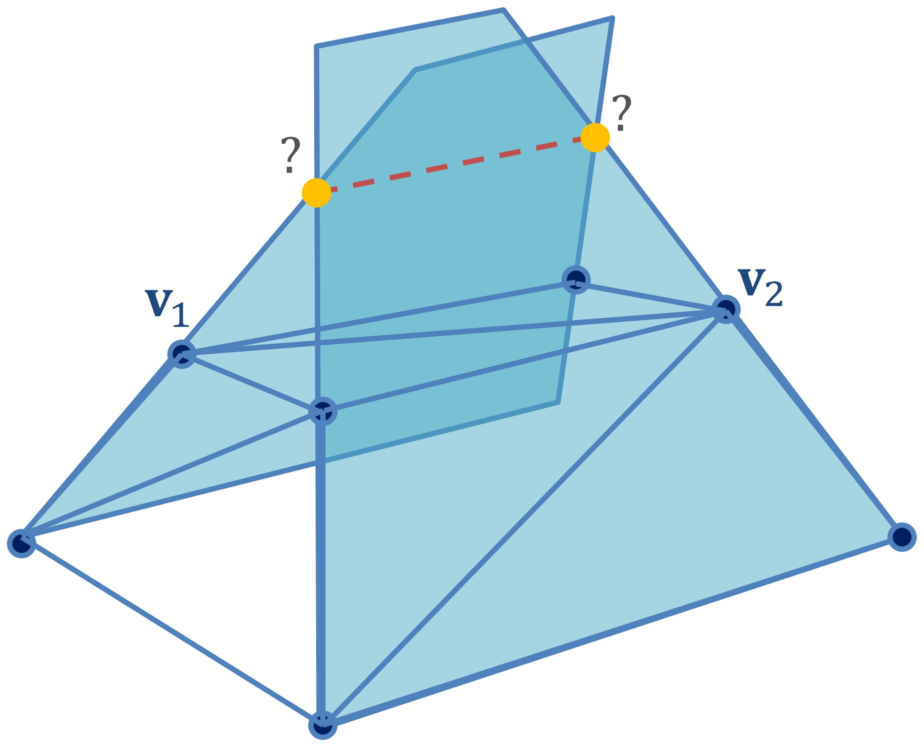
Bounding Mesh Algorithm

- If $\mathbf{Q}_1 + \mathbf{Q}_2$'s minimum point is conservative, it is the new $\bar{\mathbf{v}}$.
- Otherwise, $\bar{\mathbf{v}}$ is the closest conservative minimum point on a plane, or…
- … on the intersection of a pair of planes…



Bounding Mesh Algorithm

- ... Or, $\bar{\mathbf{v}}$ is the closest conservative point of intersection of three planes.
- Worst-case, we compute and test $1 + n + \binom{n}{2} + \binom{n}{3} = O(n^3)$ points, for n incident faces.

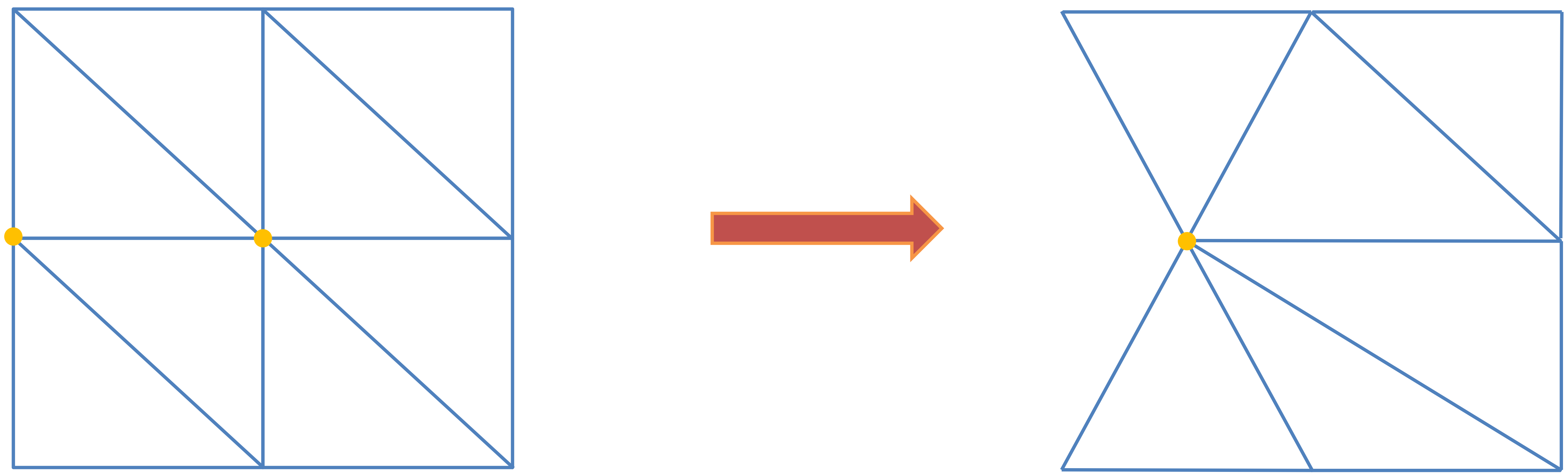


Quick and Dirty Ranking

- The contraction error is typically computed many times before the edge is contracted.
- In conservative decimation, computing the exact contraction error is expensive!
- Quick and dirty ranking of contraction candidates uses the unconstrained error.
- First-ranking edge is evaluated for a conservative vertex and possibly discarded.

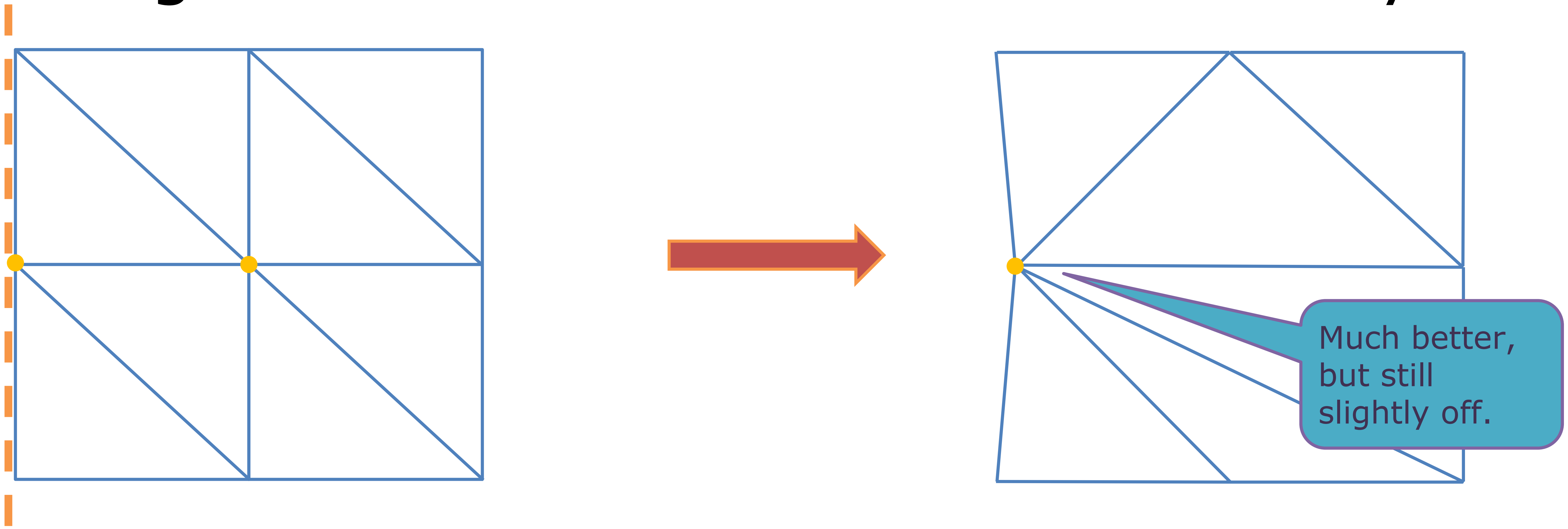
Mesh Boundaries

- Vertices at mesh boundaries tend to wander along the surface away from the boundary.



Mesh Boundaries

- Garland et al. suggest adding a virtual plane orthogonal to the surface at the boundary.



Mesh Boundaries

- Imposing hard constraints on boundary vertices keeps them from wandering.
- Conservative mesh decimation uses constrained solvers for planes and lines.
- We use the same solvers for constraining boundary vertices.
- Edges may have up to two constraint planes.

Patch Stitching

- Patch boundaries are likely to show cracks due to differences in height.
- These cracks subvert the purpose of using occluders since covered objects bleed through.
- Patch boundaries are stitched by adding vertical filler triangles.

Tighter Error Bound

- $\mathbf{Q}_1 + \mathbf{Q}_2$ is not the tightest upper bound for the minimum squared distance.
- There are better ways to construct a $\mathbf{Q}$, such that $\mathbf{Q} \geq \mathbf{Q}_1$ and $\mathbf{Q} \geq \mathbf{Q}_2$.
- Better suited if you want to decimate down to a given maximum error rather than a set number of polygons.

References

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- M. Garland and P. S. Heckbert. *Surface simplification using quadric error*. Proc. *ACM SIGGRAPH*, July 1997.
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Thanks!

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